Exercises discussed in class:

1. Show that $\text{orth}_{\mathbf{a}}\mathbf{b} = \mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$ is orthogonal to $\mathbf{a}$ (Section 12.3: Exercise 45).

2. Show the following (cf. Section 12.3: Exercise 61, 62, 63).
   (a) $\mathbf{a} \cdot \mathbf{b} \le |\mathbf{a}| \cdot |\mathbf{b}|$ (and consequently $|\mathbf{a} \cdot \mathbf{b}| \le |\mathbf{a}| \cdot |\mathbf{b}|$)
   (b) $|\mathbf{a} + \mathbf{b}|^2 \le (|\mathbf{a}| + |\mathbf{b}|)^2$ (and consequently $|\mathbf{a} + \mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$)
   (c) $|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2)$

3. Let $\mathbf{u}$ and $\mathbf{v}$ be two orthogonal unit vectors. Show the following (cf Section 12.4: Exercise 50).
   (a) $\text{comp}_{\mathbf{u}}[\mathbf{a} \times (\mathbf{u} \times \mathbf{v})] = \mathbf{a} \cdot \mathbf{v}$ [Hint: Apply $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{u} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{u})$ and $(\mathbf{u} \times \mathbf{v}) \times \mathbf{u} = \mathbf{v}$]
   (b) $\text{comp}_{\mathbf{v}}[\mathbf{a} \times (\mathbf{u} \times \mathbf{v})] = -\mathbf{a} \cdot \mathbf{u}$ [Hint: Use $(\mathbf{u} \times \mathbf{v}) \times \mathbf{v} = -\mathbf{u}$]
   (c) $\mathbf{a} \times (\mathbf{u} \times \mathbf{v}) = (\mathbf{a} \cdot \mathbf{v})\mathbf{u} - (\mathbf{a} \cdot \mathbf{u})\mathbf{v}$
   (d) If $\mathbf{b}$ and $\mathbf{c}$ are orthogonal then $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

4. Let $\mathbf{b}' = \text{proj}_{\mathbf{b}}\mathbf{c}$ and $\mathbf{c}' = \text{orth}_{\mathbf{b}}\mathbf{c}$. Show the following (cf Section 12.4: Exercise 50).
   (a) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \times (\mathbf{b} \times \mathbf{c}')$ [Hint: Use $\mathbf{c} = \mathbf{b}' + \mathbf{c}'$ and $\mathbf{b} \times \mathbf{b}' = \mathbf{0}$]
   (b) $(\mathbf{a} \cdot \mathbf{c}')\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}' = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ [Hint: Use $\mathbf{c} = \gamma\mathbf{b} + \mathbf{c}'$ with $\gamma = \dfrac{\mathbf{b} \cdot \mathbf{c}}{|\mathbf{b}|^2}$]
   (c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Key formulas to memorize: Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ with angle θ between $\mathbf{a}$ and $\mathbf{b}$.

1. (Equation of sphere centered at $\mathrm{P}(a, b, c)$) $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

2. (Dot product) $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta$

3. (Cross product) $\mathbf{a} \times \mathbf{b} = \left\langle \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right\rangle$ where $\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$

4. (Unit vector to $\mathbf{a}$) $\mathbf{u} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$ with $|\mathbf{a}|^2 = \mathbf{a} \cdot \mathbf{a}$

5. (Scalar and vector projection of $\mathbf{b}$ onto $\mathbf{a}$) $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$ and $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$

6. $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| \cdot |\mathbf{b}| \cdot |\sin\theta|$

Online quiz No.1: Start online quiz at the course website.