Exercises discussed in class:

1. Show that

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

is the distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ (Section 12.5: Exercise 75).

2. Prove the following formulas

(a)
$$\frac{d}{dt}|\mathbf{r}(t)| = \frac{\mathbf{r}(t) \cdot \mathbf{r}'(t)}{|\mathbf{r}(t)|}$$
 (Section 13.2: Exercise 57).
(b) $\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)] = \mathbf{r}(t) \times \mathbf{r}''(t)$ (Section 13.2: Exercise 55)

3. Prove the following formulas.

(a)
$$\frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$$

(b)
$$\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$$

Exercises by Maxima/wxMaxima: The following web pages provide wxMaxima demonstration for each exercise.

- 1. Vectors and Their Operations (https://vps63.heliohost.us/e-math/2110/page3.html)
 - (a) Find the plane that contains the line of intersection of x z = 1 and y + 2z = 3 and is perpendicular to x + y 2z = 1 (Section 12.5: Exercise 40).
 - (b) Let $\langle 1, 1, 0 \rangle + t \langle 1, -1, 2 \rangle$ and $\langle 2, 0, 2 \rangle + s \langle -1, 1, 0 \rangle$ be two lines.
 - i. Find the point of the intersection.
 - ii. Find a linear equation of the plane that contains these lines. (Section 12.5: Exercise 64.)
- 2. Drawing Space Curves (https://vps63.heliohost.us/e-math/2110/page4.html)
 - (a) Graph the curve $\langle t, t^2, e^{-t} \rangle$.
 - (b) Graph the curve $\langle e^{-t} \cos 5t, e^{-t} \sin 5t, e^{-t} \rangle$.
 - (c) Graph the curve $\langle \cos t, \sin t, \ln t \rangle$.
- 3. Differentiate Vector Functions (https://vps63.heliohost.us/e-math/2110/page5.html)
 - (a) Find the unit tangent vector to $\langle 4\sqrt{t}, t^2, t \rangle$ at t = 1.
 - (b) Find the equation of the tangent line (itself parametrized with s) to the curve $\langle e^t, te^t, te^{t^2} \rangle$ (parametrized by t) at t = 0.
 - (c) Integrate the curve $\langle 0, 4/(1+t^2), 2t/(1+t^2) \rangle$ from t = 0 to t = 1.
 - (d) Differentiate the cross product of $\langle \sin t, \cos t, t \rangle$ and $\langle t, \cos t, \sin t \rangle$.

Key formulas to memorize:

- 1. $x = x_0 + at$; $y = y_0 + bt$; $z = z_0 + ct$ [Line through (x_0, y_0, z_0) and parallel to $\mathbf{a} = \langle a, b, c \rangle$]
- 2. $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ [Plane through (x_0, y_0, z_0) and orthogonal to $\mathbf{n} = \langle a, b, c \rangle$]
- 3. ax + by + cz + d = 0 [Plane orthogonal to $\mathbf{n} = \langle a, b, c \rangle$]
- 4. $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} \text{ [Distance from a point } (x_1, y_1, z_1) \text{ to a plane } ax + by + cz + d = 0]$ 5. $\langle x, y, z \rangle = \mathbf{r}(s_0) + t \, \mathbf{r}'(s_0) \text{ [Tangent line to } \mathbf{r}(s) \text{ at } s = s_0]$ 6. $\frac{d}{dt} [\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$ 7. $\frac{d}{dt} [\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$

Online quiz No.2: Start online quiz at the course website.