Exercises by Maxima/wxMaxima. You are encouraged to work on exercises by downloading Maxima/wxMaxima files (.wmx) and running them on your own computer. The following web pages provide wxMaxima operations relevant to the topics coverned by this assignment.

1. Surface and Contour (`https://vps63.heliohost.us/e-math/2110/page8.html`)
2. Partial Derivatives (`https://vps63.heliohost.us/e-math/2110/page9.html`)

Work through examples presented in the class, and complete the following exercises.

1. Drawing surface and contour (`e-math/2110/exercise14-1.wxm`)
   (a) Draw the graph of $z = 1/(1 + x^2 + y^2)$.
   (b) Draw the graph and the contour map for $z = e^x \cos y$.
   (c) Draw the graph and the contour map for $z = (x - y)/(1 + x^2 + y^2)$.
2. Finding partial derivatives (`e-math/2110/exercise14-3.wxm`)
   (a) Find the first partial derivatives of $\tan xy$.
   (b) Find the first partial derivatives of x^y.
   (c) Find the second partial derivatives of $\sin^2(mx + ny)$.
   (d) Find the second partial derivatives of $xy/(x - y)$.
   (e) Find the second partial derivatives of e^{xe^y}
3. Tangent planes and differentials (`e-math/2110/exercise14-4.wxm`)
   (a) Find the tangent plane to the surface $f(x, y) = 3(x - 1)^2 + 2(y + 3)^2 + 7$ at $x = 2$ and $y = -2$.
   (b) Find the linearization of $f(x, y) = \sqrt{x + e^{4y}}$ at $x = 3$ and $y = 0$.
   (c) Find the differential of $z = y \cos xy$.
4. Chain rules and implicit differentiation (`e-math/2110/exercise14-5.wxm`)
   (a) Find the first derivative of $z = \cos(x + 4y)$, $x = 5t^4$, $y = 1/t$.
   (b) Find the first derivative of $w = \log \sqrt{x^2 + y^2 + z^2}$, $x = \sin t$, $y = \cos t$, $z = \tan t$.
   (c) Find the first partial derivatives of $z = \arcsin(x - y)$, $x = s^2 + t^2$, $y = 1 - 2st$.
   (d) Find dy/dx for the implicit equation $y^5 + x^2y^3 = ye^{x^2} + 1$.
   (e) Find the first partial derivative of z with respect to x and y for the implicit equation $xyz = \cos(x + y + z)$.
   (f) Let $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$.
      i. Find $\dfrac{\partial z}{\partial r}$, $\dfrac{\partial z}{\partial \theta}$, and $\dfrac{\partial^2 z}{\partial r \partial \theta}$ (Section 14.5: Exercise 54).
      ii. Show that $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = \left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2$ (Section 14.5: Exercise 49)

iii. Show that $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=\dfrac{\partial^2 z}{\partial r^2}+\dfrac{1}{r^2}\dfrac{\partial^2 z}{\partial \theta^2}+\dfrac{1}{r}\dfrac{\partial z}{\partial r}$ (Section 14.5: Exercise 55)

(g) Let $z=f(x,y)$ where $x=e^s\cos t$ and $y=e^s\sin t$.

i. Show that $\left(\dfrac{\partial z}{\partial x}\right)^2+\left(\dfrac{\partial z}{\partial y}\right)^2=e^{-2s}\left[\left(\dfrac{\partial z}{\partial s}\right)^2+\left(\dfrac{\partial z}{\partial t}\right)^2\right]$ (Section 14.5: Exercise 50)

ii. Show that $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=e^{-2s}\left[\dfrac{\partial^2 z}{\partial s^2}+\dfrac{\partial^2 z}{\partial t^2}\right]$ (Section 14.5: Exercise 52)

Online quiz No.4: Complete online quiz at the course website.

Key formulas to memorize:

1. $z-z_0=f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$ [tangent plane at (x_0,y_0,z_0)]
2. $dz=f_x(x,y)dx+f_y(x,y)dy$ [total differential]
3. $\dfrac{dz}{dt}=f_x(x,y)\dfrac{dx}{dt}+f_y(x,y)\dfrac{dy}{dt}$
   [chain rule of derivative for $z=f(x,y)$ when $x=g(t)$ and $y=h(t)$]
4. $\dfrac{\partial z}{\partial s}=f_x(x,y)\dfrac{\partial x}{\partial s}+f_y(x,y)\dfrac{\partial y}{\partial s}$; $\dfrac{\partial z}{\partial t}=f_x(x,y)\dfrac{\partial x}{\partial t}+f_y(x,y)\dfrac{\partial y}{\partial t}$
   [chain rule of partial derivatives for $z=f(x,y)$ when $x=g(s,t)$ and $y=h(s,t)$]