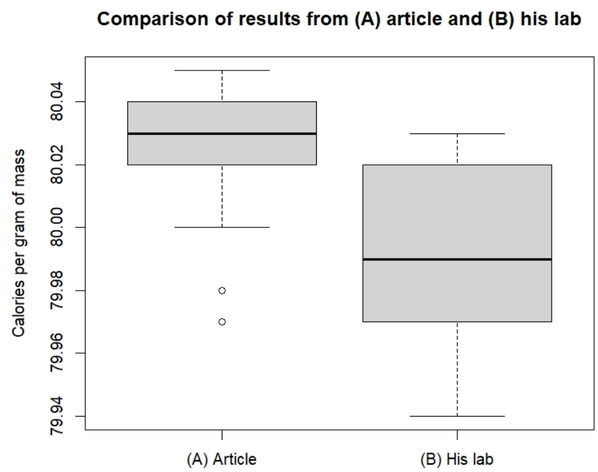
Problem 7. A researcher read an article about the latent heat of a particular material and decided to reproduce the same experiment discussed in the article. The following table and boxplots compare the data set from the article and the data produced by his lab. He initially thought that the sample mean $\bar{X} = 80.02$ from the article data is almost the same as the sample mean $\bar{Y} = 79.99$ from his lab, and therefore that he successfully reproduced the result obtained by the article. But he also notice that the boxplots from the respective data are somewhat different.

Article	His lab
79.98	80.02
80.04	79.94
80.02	79.98
80.04	79.97
80.03	80.00
80.03	80.03
80.04	
79.97	
80.05	
80.03	
80.02	
80.00	
80.02	



(a) $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 \neq \mu_2$

Note that there is no hypothesis test for $H_A : \mu_1 = \mu_2$.

(b) We can calculate the pooled variance $S_p^2 \approx 0.00073$. The test statistic is

$$T = \frac{\bar{X} - \bar{Y}}{S_p\sqrt{1/13 + 1/6}} = 2.3,$$

and the 95% confidence interval for $\mu_1 - \mu_2$ is $(0.003, 0.059)$.

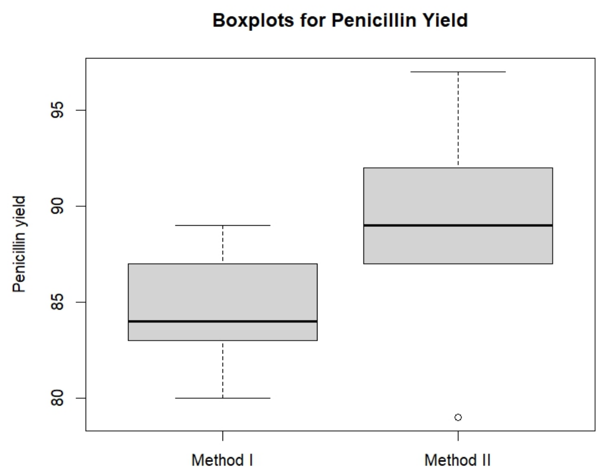
(c) In the Neyman-Pearson framework we cannot prove that the two means are equal, and the confidence interval for the difference $\mu_1 - \mu_2$ suggests that they could be different. Since $t_{0.05/2,17} = 2.11 < T = 2.3 < t_{0.01/2,17} = 2.90$, we can reject H_0 if $\alpha = 0.05$, but cannot reject it if $\alpha = 0.01$. In fact, the 99% confidence interval $(-0.008, 0.070)$ suggests that the difference $\mu_1 - \mu_2$ could be zero. It does not mean that we are confident that $\mu_1 = \mu_2$ with probability 99%.

(d) The result rather indicates that the two means are different with the choice of significance level $\alpha = 0.05$, while there is no such evidence if you choose $\alpha = 0.01$. Thus, we cannot justify that the lab result has reproduced the article result, and the difference is somewhat significant.

Problem 8. On the first day of a new job, you were asked to accompany the executive to a biomedical research firm. This firm developed a new penicillin manufacturing process and offered your company an exclusive right to use this method. The decision to use this method is a serious investment for the company. But if this method can produce more penicillin per manufacturing unit, it will bring in a huge profit. The executive asked his staff for an investigation on how well new method works compared to the current one. Here is the summary of a report produced by his staff:

> To determine the effect of the new method on the yield of penicillin, the data were collected for five types of base blend (B1 to B5) to produce penicillin. "Method I" refers to the company's current process, and "Method II" to the process newly developed by the biomedical firm.

Blend	Method I	Method II
B1	89	97
B2	84	92
B3	83	87
B4	87	89
B5	80	79



> The above boxplots for penicillin yield suggests that the new method works better, as the firm claimed. For statistical inference, the null hypothesis becomes
>
> $$H_0 : \mu_1 = \mu_2 \quad \text{versus} \quad H_A : \mu_1 < \mu_2$$
>
> where μ_1 and μ_2 are the population means for Method I and Method II, respectively. An inference on two independent samples was considered and the p-value $p^* = 0.1234$ was obtained. Thus, the null hypothesis cannot be rejected, indicating that there is not sufficient evidence to support the biomedical firm's claim.

As presented in the summary, the report was not positive to the biomedical firm's new method, and the executive should decline the firm's offer. But he cannot believe that his staff could not find a sufficient evidence. So he asked you what you think. You, unaware of the seriousness of the meeting, did not bring your laptop PC, but happened to have the t-distribution table. Try to do your own analysis with pen and paper, and give your opinion to your boss.

(a) Let X_i be the penicillin yield from i-th base blend in Method I, and let Y_i be the penicillin yield from i-th base blend in Method II. Since the factor of base blend unfairly influences the outcome, $X_1, \ldots, X_5$ are not iid random variables, and so are $Y_1, \ldots, Y_5$. However, the difference $Z_i = X_i - Y_i$ can cancel the influence of base blend so that $Z_1, \ldots, Z_5$ are iid random variables.

Let μ be the population mean for $Z_1, \ldots, Z_5$.

The test for the comparison of two groups assumes that X_i and Y_i are independent, and the failure of this requirement would lead to a wrong conclusion based on an inaccurate sampling distribution. But $Z_1, \ldots, Z_5$ are independent, and the test for "$H_0 : \mu = 0$ vs. $H_0 : \mu < 0$" is appropriate; thus, it would be accurate.

(b) The test statistic is $T = \dfrac{\bar{Z}}{S/\sqrt{5}} = -2.41$, and we can observe

$$-t_{0.01,4} = -3.747 < T = -2.41 < -t_{0.05,4} = -2.132.$$

(c) The result suggests that there is modestly significant difference between Method I and Method II. We can conclude that the yield is higher in Method II when we choose the significance level $\alpha = 0.05$, but we cannot make the same conclusion if the significance level $\alpha = 0.01$ is chosen.