$$P(A) = \frac{\text{\# of outcomes in } A}{\text{\# of outcomes in } \Omega}$$ How to count?

Q.1 (4 points) A fair die is tossed 2 times. Then the sample space becomes

$$\Omega = \{(i, j) : i, j = 1, 2, 3, 4, 5, 6\}.$$

Let A be the event that there is at least one six, and let B be the event that there is at least one five.

(a) Express the event A as a subset of Ω.

$A = \{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6)\}$

(b) Express the event $A^c \cap B$ as a subset of Ω.

$A^c \cap B = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (1, 5), (2, 5), (3, 5), (4, 5)\}$

(c) Find $P(A^c \cap B)$ and $P(A^c)$.

$P(A^c \cap B) = \frac{9}{36} = \frac{1}{4}$ and $\underline{P(A^c) = 1 - P(A)} = 1 - \frac{11}{36} = \frac{25}{36}$

Q.2 (2 points) Let A and B be events. Suppose that the sample space Ω satisfies $\Omega = A \cup B$, and that $P(A) = 0.7$ and $P(B) = 0.6$. Find $P(A \cap B)$.

Since $1 = P(A \cup B) = P(A) + P(B) - P(A \cup B) = 1.3 - P(A \cup B)$, we obtain $P(A \cap B) = 0.3$.

Q.3 (2 points) Discuss what is common between the following statements?

- The coefficient of x^3y^4 in the expansion of $(x + y)^7$;
- the number of patterns placing three red blocks and four green blocks all in a line.

The expansion of $(x + y)^7$ forms $3x$'s and $4'y$'s in different orders (or patterns) whose sum determines the coeffiecient of x^3y^4. In order to find a particular "pattern," we choose 3 places for x's from 1st to seventh place and fill in with y's for the rest of places. Exactly in the same way we can create a "pattern" for three red blocks and four green blocks. The number of patterns is given by $\binom{7}{3} = 35$.

Q.4 (2 points) From a group of 7 students including Amanda and Brad, how many different ways to form a three-member committee in which Amanda and Brad do not serve together.

$\binom{7}{3} - 5 = 30.$