Q.1 (3 points) A fair die is tossed 2 times. Then the sample space becomes

$$\Omega = \{(i, j) : i, j = 1, 2, 3, 4, 5, 6\}.$$

Let A be the event that there is at least one six, and let B be the event that there is at least one five.

(a) Express the event A as a subset of Ω.

$A = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6),(1,6),(2,6),(3,6),(4,6),(5,6)\}$

(b) Express the event $A^c \cap B$ as a subset of Ω.

$A^c \cap B = \{(5,1),(5,2),(5,3),(5,4),(5,5),(1,5),(2,5),(3,5),(4,5)\}$

(c) Find $P(A^c \cap B)$.

Similarly to homework problem 2(e) we obtain $P(A^c \cap B) = \frac{9}{36} = \frac{1}{4}$.

Q.2 (1 points) What is common between the following statements? Discuss it.

- The coefficient of x^3y^4 in the expansion of $(x + y)^7$;
- the number of patterns placing three red blocks and four green blocks all in a line.

The expansion of $(x + y)^7$ forms $3x$'s and $4'y$'s in different orders (or patterns) whose sum determines the coefficient of x^3y^4. In order to find a particular "pattern," we choose 3 places for x's from 1st to 7th place and fill in with y's for the rest of places. Exactly in the same way we can create a "pattern" for three red blocks and four green blocks. The number of patterns is given by $\binom{7}{3} = 35$.

Q.3 (2 points) From a group of 7 students including Amanda and Brad, we form a three-member committee. What is the probability that Amanda and Brad do not serve together?

Similarly to homework problem 6, there are 5 different three-member committee in which Amanda and Brad serve together. Thus, we obtain $\dfrac{\binom{7}{3} - 5}{\binom{7}{3}} = 30/35 = 6/7$.

Q.4 (4 points) Urn 1 has three red balls and two white balls, and urn 2 has one red balls and five white balls. Let A be the event that urn 1 is chosen to draw a ball, and let B be the event that a red ball is drawn. Suppose that we know $P(A) = \frac{1}{4}$. Then answer the following questions.

(a) Find $P(B)$.

Instead of tossing a coin to decide which urn to choose, you are given $P(A) = \frac{1}{4}$. Otherwise, it goes exactly as in homework problem 8.

$$P(B|A)P(A) + P(B|A^c)P(A^c) = \left(\frac{3}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{3}{4}\right) = \frac{11}{40}$$

(b) Find the *conditional probability* $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{(3/5)(1/4)}{(11/40)} = \frac{6}{11}$$