Conditional probability: $P(B|A) = \frac{P(B\cap A)}{P(A)}$

 $P(B\cap A) = P(B|A)P(A)$

Q.1 (4 points) Urn 1 has three red balls and two white balls, and urn 2 has one red balls and five white balls. Let A be the event that a ball is drawn from urn 1, and let B be the event that a red ball is drawn. Suppose that we know $P(A) = \frac{1}{4}$. Then answer the following questions.

(a) Find $P(B)$.

Instead of tossing a coin to decide which urn to choose. You are given $P(A) = \frac{1}{4}$. Otherwise, it goes exactly as in Problem 8.

$$P(B|A)P(A) + P(B|A^c)P(A^c) = \left(\frac{3}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{3}{4}\right) = \frac{11}{40}$$

(b) Find the *conditional probability* $P(A|B)$.

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{(3/5)(1/4)}{(11/40)} = \frac{6}{11}$$

Q.2 (6 points) Let B and D be independent events with $P(B) = \frac{1}{5}$ and $P(D) = \frac{1}{3}$.

(a) Find $P(B\cap D)$.

Since B and D are independent, we have $P(B\cap D) = P(B)P(D) = \frac{1}{15}$

(b) Find $P(B\cup D)$.

$P(B\cup D) = P(B) + P(D) - P(B\cap D) = \frac{1}{5} + \frac{1}{3} - \frac{1}{15} = \frac{7}{15}$

(c) Find the *conditional probability* $P(B|B\cup D)$.

Problem 7 similarly uses the definition of conditional probability. But you must also notice $B\cap(B\cup D) = B$. Then we can find

$$P(B|B\cup D) = \frac{P(B)}{P(B\cup D)} = \frac{1/5}{7/15} = \frac{3}{7}$$

Independence of A and B if $P(A\cap B) = P(A)P(B)$