Understand the difference between joint frequency function and joint density function

Q.1 (4 points) A discrete random variable X has a frequency function $p(x) = P(X = x)$

$$p(x) = \begin{cases} 3c & \text{if } x = 0; \\ (6-x)c & \text{if } x = 1, 2, 3, \end{cases}$$

with constant c.

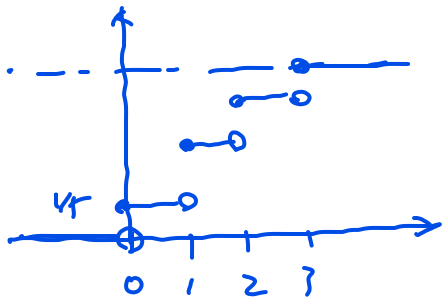
(a) Determine the constant c, and find $P(\frac{1}{2} < X < \frac{7}{2})$.

Since $\sum_{x=0}^{3} p(x) = 15c = 1$, we obtain $c = \frac{1}{15}$. Then we find

$$P(\tfrac{1}{2} < X < \tfrac{7}{2}) = p(1) + p(2) + p(3) = \tfrac{4}{5}$$

(b) Sketch the graph of the cdf $F(x)$. $= P(X \le x)$

$$F(x) = \begin{cases} 0 & \text{if } x < 0; \\ 1/5 & \text{if } 0 \le x < 1; \\ 8/15 & \text{if } 1 \le x < 2; \\ 4/5 & \text{if } 2 \le x < 3; \\ 1 & \text{if } 3 \le x. \end{cases}$$



Q.2 (6 points) Let X and Y be random variables. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} x + y & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1; \\ 0 & \text{otherwise.} \end{cases}$$

X and Y are continuous

(a) Find $P(X \le \frac{1}{2}, Y \le \frac{1}{2})$.

$$P(X \le \tfrac{1}{2}, Y \le \tfrac{1}{2}) = \int_0^{1/2} \int_0^{1/2} (x+y)\, dx\, dy = \frac{1}{8}$$

(b) Find the marginal density function for X.

$$f_X(x) = \int_0^1 (x+y)\, dy = x + \frac{1}{2}.$$

(c) Find the conditional density function of Y given $X = x$.

$$f_{Y|X}(y|x) = \frac{x+y}{x+1/2} \text{ for } 0 \le y \le 1.$$