

**Q.1** (5 points) A bowl contains 5 red chips and 10 blue chips, and a person randomly picks 3 chips from the bowl (a) with replacement, or (b) without replacement. Let  $X$  be the number of red chips drawn. Answer the following questions.

- (a) When each draw is made with replacement, describe the exact distribution for  $X$ . Find the frequency function  $p(k)$  for  $X$ .

It is a binomial distribution with  $n = 3$  and  $p = 1/3$ .

$$p(k) = \binom{3}{k} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{3-k} \text{ for } k = 0, 1, 2, 3.$$

- (b) When each draw is made without replacement, describe the exact distribution for  $X$ . Find the frequency function  $p(k)$  for  $X$ .

It is the hypergeometric distribution

$$p(k) = \frac{\binom{5}{k} \binom{10}{3-k}}{\binom{15}{3}} \text{ for } k = 0, 1, 2, 3.$$

- (c) Find  $E[X]$  for (a) and (b) above. Are they different?

(a)  $E[X] = np = 1$ ; (b)  $E[X] = \frac{\binom{5}{1}\binom{10}{2}}{\binom{15}{3}} = 1$ . Therefore, they are the same.

**Q.2** (5 points) Suppose that a random variable  $X$  has a binomial distribution with parameter  $(n, p)$ . Answer the following questions.

- (a) Find  $E\left[\frac{X}{n}\right]$  in terms of  $p$

$$E\left[\frac{X}{n}\right] = p.$$

- (b) Find  $\text{Var}\left(\frac{X}{n}\right)$  in terms of  $n$  and  $p$ .

$$\text{Var}\left(\frac{X}{n}\right) = \left(\frac{1}{n}\right)^2 \text{Var}(X) = \frac{p(1-p)}{n}.$$

- (c) Find  $E[X^2]$  in terms of  $n$  and  $p$ .

$$E[X^2] = \text{Var}(X) + (E[X])^2 = np(1-p) + (np)^2.$$