Your name: _____
(first) (last)

All the output for the assignment should be printed and attached to this sheet.

Question 1. Define the square matrix A whose entries are defined by

$$a_{ij} = \begin{cases} 2 & \text{if } i = j; \\ 5 & \text{otherwise.} \end{cases}$$

1. Write a Matlab/Octave function which generates the above $n \times n$ square matrix A.

```
function A = aamat(n)
% aamat(n) creates the n-by-n square matrix.
...
```

2. For each $n = 2, 3, 4$, calculate $\det A$, $\det A^2$, and $\det(2A)$. Confirm the relationship between them. That is, $\det A^2 = (\det A)^2$ and $\det(2A) = 2^n \det(A)$.

```
> det(A)
> det(A^2)
> det(2 * A)
```

3. The determinant of the above $n \times n$ square matrix A can be explicitly written by

$$\det A = (-\alpha)^{n-1}[2 + \beta(n-1)]. \qquad (*)$$

Complete the following tasks, and find the integers α and β which correctly predict the value of $\det A$.

   (a) Create a Matlab/Octave function to calculate (*) for any choice of α, β, and n.

```
function k = aadet(a,b,n)
% It returns the value predicted by the formula for det(A).
k = (-a)^(n-1)*(2+b*(n-1));
```

   (b) For $n = 2, 3, 4$, calculate $\det A$ and the formula (*) using your best guess for α and β from $2, 3$ or 5. Repeat it until you find the correct values α and β.

   (c) For each $n = 6, 8, 10$, use the values α and β you found in (b), and confirm that the formula (*) correctly predicts the value of $\det A$

4. (A bonus question) Can you prove (*) for $n = 2, 3, 4$, and deduce α and β?

Question 2. Define the square matrix B whose entries are defined by

$$b_{ij} = \max(1, 3(\min(i, j) - 1)).$$

1. Write a Matlab/Octave function that generates the $n \times n$ square matrix B.

```
function B = bbmat(n)
% bbmat(n) creates the n-by-n square matrix.
for i = 1:n
  for j = 1:n
    B(i,j) = max(1,3*(min(i,j)-1));
  end
end
```

   Use the Matlab/Octave function `min()` and `max()`. Here you can use `max()` in the following ways:

```
> max(5,2)
> x = [6 9 7 3]
> max(x)
```

   The function `min()` can be used in a similar manner.

2. For each $n = 3, 4, 5$, calculate $\det B$, and make your own conjecture of the entire formula for $\det B$ with any choice of n.

3. Write a Matlab/Octave function for the formula you guessed.

```
function k = bbdet(n)
% It returns the value predicted by the formula for det(B).
if(n <= 2)
  k = factorial(n);
else
  k = ...
end
```

   where the function `factorial(n)` calculates the factorial $n!$.

4. Compare the actual value of $\det B$ and the output of your function for $n = 6, 8, 10$, and confirm that your conjecture is correct.

5. (A bonus question) Can you prove the formula for $\det B$ when $n = 3, 4, 5$?