

Your name: _____
(first) (last)

All the calculations for the assignment should be attached to this sheet.

Question 1. The following computer output was produced by Matlab/Octave. Then answer to the following questions.

```
> eig(A)
ans =
    2.0000
    3.0000
    3.0000
    2.0000
    2.0000

> rref([(A - 2 * eye (5)) zeros (5, 1)])
ans =
    1.00000    0.00000   -4.00000    2.00000    0.50000    0.00000
    0.00000    1.00000    1.00000   -1.00000   -0.50000    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000

> rref([(A - 3 * eye (5)) zeros (5, 1)])
ans =
    1.00000    0.00000    0.00000    0.00000    0.33333    0.00000
    0.00000    1.00000    0.00000    2.00000   -2.00000    0.00000
    0.00000    0.00000    1.00000    0.00000   -0.33333    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000
```

(a) Find the basis for the eigenspace corresponding to the eigenvalue $\lambda = 2$ of A .

(b) Find the basis for the eigenspace corresponding to the eigenvalue $\lambda = 3$ of A .

Question 2. The following computer output was produced by Matlab/Octave. Then answer to the following questions.

```
> eig(A)
ans =
    24.0000
   -4.0000
    1.0000
   -4.0000

> rref([(A - eye (4)) zeros (4, 1)])
ans =
    1.00000    0.00000    0.00000   -1.00000    0.00000
    0.00000    1.00000    0.00000   -1.00000    0.00000
    0.00000    0.00000    1.00000    2.00000    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000

> rref([(A + 4 * eye (4)) zeros (4, 1)])
ans =
    1    0    2    1    0
    0    1    0    0    0
    0    0    0    0    0
    0    0    0    0    0

> rref([(A - 24 * eye (4)) zeros (4, 1)])
ans =
    1.00000    0.00000    0.00000   -5.60000    0.00000
    0.00000    1.00000    0.00000   -5.60000    0.00000
    0.00000    0.00000    1.00000   -7.20000    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000
```

(a) For each eigenvalue $\lambda = 1, -4, 24$ of A , find the basis for the corresponding eigenspace.

(b) Find a matrix P and a diagonal matrix D so that we have $A = PDP^{-1}$. Then find A .

(c) (Bonus question) Let $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = A^k \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Express v_1 in terms of k .