Theoretical Results on Sampling Distributions

Normality assumption of random sample. We may assume that a random sample consists of independent normal random variables $X_1, \ldots, X_n$ with parameter (μ, σ^2); in short we write

$$X_i \overset{\text{iid}}{\sim} N(\mu, \sigma^2), \quad i = 1, \ldots, n, \tag{7.1}$$

where "iid" stands for "independent and identically distributed." Then the sample mean

$$\bar{X} := \frac{1}{n} \sum_{i=1}^{n} X_i$$

is normally distributed with parameter $(\mu, \sigma^2/n)$. The property (7.1) is called *normality assumption.*

The *t* distributions. Suppose that a random variable X has the standard normal distribution, that a random variable Y has the chi-square distribution with n degrees of freedom (which is a gamma distribution with $\alpha = n/2$ and $\lambda = 1/2$) and that X and Y are independent. Then the distribution of the random variable

$$\frac{X}{\sqrt{Y/n}}$$

is called a **t-distribution** with n degrees of freedom.

Critical values. Given a random variable X, the **critical value** for level α is defined as the value c satisfying $P(X > c) = \alpha$. Suppose that X has the t-distribution with m degrees of freedom. Then the critical value is defined as the value $t_{\alpha,m}$ satisfying

$$P(X > t_{\alpha,m}) = \alpha.$$

Since the t-distribution is symmetric, the critical value $t_{\alpha,m}$ can be equivalently given by

$$P(X < -t_{\alpha,m}) = \alpha.$$

Together we can obtain the expression

$$\begin{aligned} &P(-t_{\alpha/2,m} \le X \le t_{\alpha/2,m}) \\ &\quad = 1 - P(X < -t_{\alpha/2,m}) - P(X > t_{\alpha/2,m}) = 1 - \alpha. \end{aligned}$$

Distributions under normality assumption. Provided the normal assumption for a data set, the following properties regarding the sample mean $\bar{X}$ and the sample variance S^2 are extremely useful in statistical applications.

(A) The sample mean $\bar{X}$ is normally distributed with parameter $(\mu, \sigma^2/n)$ and the random variable (7.2) has the standard normal distribution.

$$\frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \sim N(0, 1) \tag{7.2}$$

Distributions under normality assumption. (B) The random variable (7.3) is distributed as the chi-square distribution with $(n-1)$ degrees of freedom, and is independent of the random variable (7.2).

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1} \tag{7.3}$$

(C) The random variable (7.4) has the t-distribution with $(n-1)$ degrees of freedom, where S is the sample standard deviation $S = \sqrt{S^2}$.

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}}{\sqrt{\frac{(n-1)S^2/\sigma^2}{n-1}}} \sim t_{n-1} \tag{7.4}$$

Confidence Interval for Mean

Parameter of underlying distribution. A random sample

$$X_1, \ldots, X_n \tag{7.5}$$

is regarded as "independent and identically distributed" random variables (called iid r.v.'s in short) governed by an underlying probability density function $f(x;\theta)$. A value θ represents the characteristics of the underlying distribution, and is called a **parameter**. Under the normal assumption the underlying distribution is the normal distribution with (μ, σ^2). Then the values μ and σ^2 are the parameters.

Statistics and estimates. Since $X_1, \ldots, X_n$ are random variables, the sample mean $\bar{X}$ also becomes a random variable. A random variable constructed from the random sample is called a **statistic**. A **point estimate** is a statistic $\hat{\theta}$ which is a "best guess" for the true value θ. Under the normal assumption the underlying distribution is the normal distribution with (μ, σ^2), and the sample mean

$$\bar{X} := \frac{1}{n}\sum_{i=1}^{n} X_i$$

is a best guess of the parameter μ.

Confidence interval. Suppose that (7.5) are normally distributed. Then the random variable

$$\frac{\bar{X}-\mu}{S/\sqrt{n}}$$

has the t-distribution with $(n-1)$ degrees of freedom. By using the critical value $t_{\alpha/2,n-1}$, we can obtain the probability

$$P\left(-t_{\alpha/2,n-1} \le \frac{\bar{X}-\mu}{S/\sqrt{n}} \le t_{\alpha/2,n-1}\right) = 1-\alpha.$$

Confidence interval, continued. The event in the above probability can be also expressed as

$$\left\{\bar{X} - \frac{t_{\alpha/2,n-1}S}{\sqrt{n}} \le \mu \le \bar{X} + \frac{t_{\alpha/2,n-1}S}{\sqrt{n}}\right\}.$$

This indicates that the population mean μ is in the interval

$$\left(\bar{X} - \frac{t_{\alpha/2,n-1}S}{\sqrt{n}},\ \bar{X} + \frac{t_{\alpha/2,n-1}S}{\sqrt{n}}\right) \tag{7.6}$$

with probability $(1-\alpha)$. The interval (7.6) is called the **confidence interval** of level $(1-\alpha)$. Even if (7.5) are not normally distributed, it is sensible to use critical values from the t-distribution.

Example 1. A random sample of $n = 30$ milk containers is selected, and their milk contents are weighed. The data $X_1, \dots, X_n$ can be used to investigate the unknown population mean of the milk container weights. Suppose that we calculate $\bar{X} = 2.073$ and $S = 0.071$ from the actual data with $n = 30$. Construct a 95% two-sided confidence interval for the population mean.

Solution. Choose $\alpha = 0.05$. Then the critical value $t_{0.025,29} = 2.045$ gives the confidence interval

$$\left(2.073 - \frac{2.045 \times 0.071}{\sqrt{30}},\ 2.073 + \frac{2.045 \times 0.071}{\sqrt{30}}\right) = (2.046,\ 2.100)$$

of level 0.95 (or, of level 95%).

Concept of Statistical Tests

Neyman-Pearson framework. Suppose that a researcher is interested in whether the new drug works. The process of determining whether the outcome of the experiment points to "yes" or "no" is called **hypothesis testing**. A widely used formalization of this process is due to Neyman and Pearson. Our hypothesis is then the **null hypothesis** that the new drug has no effect —the null hypothesis is often the reverse of what we actually believe, why? Because the researcher hopes to reject the hypothesis and announce that the new drug leads to *significant* improvements. If the hypothesis is *not* rejected, the researcher announces *nothing* and goes on to a new experiment.

Hypothesis testing of population mean. Hospital workers are subject to a radiation exposure emanating from the skin of the patient. A researcher is interested in the plausibility of the statement that the population mean μ of radiation level is μ_0 —the researcher's hypothesis. Then the **null hypothesis** is

$$H_0:\ \mu = \mu_0.$$

The "opposite" of the null hypothesis, called an **alternative hypothesis**, becomes

$$H_A:\ \mu \ne \mu_0.$$

Thus, the hypothesis testing problem "H_0 versus H_A" is formed. The problem here is to whether or not to reject "H_0 in favor of H_A."

Constructing of two-sided t-test. To assess this hypothesis, the radiation levels $X_1, \dots, X_n$ were measured from n patients who had been injected with a radioactive tracer. Under the null hypothesis, the random variable

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \tag{7.7}$$

can be assumed to have the t-distribution with $(n-1)$ degrees of freedom; thus, we can obtain the exact probability

$$P\left(|T| \ge t_{\alpha/2,n-1}\right) = \alpha. \tag{7.8}$$

When α is chosen to be a small value (0.05 or 0.01, for example), it is *unlikely* that the absolute value $|T|$ is larger than the critical value $t_{\alpha/2,n-1}$. Then we say that the null hypothesis H_0 is *rejected* with **significance level** α (or, **size** α) when the observed value t of T satisfies $|t| > t_{\alpha/2,n-1}$.

Example 2. We have $\mu_0 = 5.4$ for the hypothesis, and decided to give a test with significance level $\alpha = 0.05$. Suppose that we have obtained $\bar{X} = 5.145$ and $S = 0.7524$ from the actual data with $n = 28$.

Solution. We can compute

$$T = \frac{5.145 - 5.4}{0.7524/\sqrt{28}} \approx -1.79.$$

Since $|T| = 1.79 \le t_{0.025,27} = 2.052$, the null hypothesis cannot be rejected. Thus, the evidence against the null hypothesis is not persuasive.

The p-value. The random variable $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$ is called the t-**statistic**. Having observed "$T = t$," we can calculate the p-**value**

$$p^* = P(|Y| \ge |t|) = 2 \times P(Y \ge |t|),$$

where the random variable Y has a t-distribution with $(n-1)$ degrees of freedom. Then we have the relation "$p^* < \alpha \Leftrightarrow |t| > t_{\alpha/2,n-1}$." Thus, we reject H_0 with significance level α when $p^* < \alpha$. In the above example, we can compute the p-value $p^* = 2 \times P(Y \ge 1.79) \approx 0.0847 \ge 0.05$; thus, we cannot reject H_0.

One-sided Hypothesis Tests

One-sided hypothesis test. Consider the same example of hospital workers subject to a radiation exposure. This time the researcher is interested in the plausibility of the statement that the population mean μ is less than μ_0. Then the hypothesis testing problem becomes

$$H_0:\ \mu = \mu_0 \quad \text{versus} \quad H_A:\ \mu < \mu_0.$$

(a) The same t-statistics $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$ is used as a **test statistic**. We reject H_0 with significant level α **in favor of** H_A when you find that $t < -t_{\alpha,n-1}$ for the observed value t of T. (b) Alternatively we can construct the p-value

$$p^* = P(Y \le t),$$

with Y of t-distribution having $(n-1)$ degrees of freedom. Because of the relation "$p^* < \alpha \Leftrightarrow t < -t_{\alpha,n-1}$," we can reject H_0 with significant level α when $p^* < \alpha$.

Example 3. We use the same $\mu_0 = 5.4$ for the hypothesis and choose the same significance level $\alpha = 0.05$ for the above one-sided test. Recall that $\bar{X} = 5.145$ and $S = 0.7524$ were obtained from the data with $n = 28$.

Solution. (a)We compute

$$T = \frac{5.145 - 5.4}{0.7524/\sqrt{28}} \approx -1.79.$$

Since $T = -1.79 < -t_{0.05,27} = -1.703$, the null hypothesis H_0 is rejected. Thus, the outcome is statistically significant so that the population mean μ is smaller than 5.4. (b) Alternatively, we can find the p-value $p^* = P(Y \le -1.79) \approx 0.0423 < 0.05$; thus, the null hypothesis should be rejected.

Type I error. What is the probability that we incorrectly reject H_0 when it is actually true? The probability of such an error, called the probability of **type I error**, is exactly the significant level α, as explained in the following (i) and (ii):

(i) Two-sided hypothesis testing: The probability of type I error is exactly the probability α in (7.8).

(ii) One-sided hypothesis testing: If the alternative hypothesis (which is the claim of interest) is false, we must have $\delta = \dfrac{\mu - \mu_0}{S/\sqrt{n}} \ge 0$. Therefore, $\frac{\bar{X}-\mu}{S/\sqrt{n}} = T - \delta$ has the t-distribution with $(n-1)$ degrees of freedom. Thus, the worst (that is, the largest possible) probability of type I error is given by

$$P(T \le -t_{\alpha,n-1}) \le P(T - \delta \le -t_{\alpha,n-1}) = \alpha.$$

One-sided hypothesis testing with "$H_A : \mu > \mu_0$." Consider the hypothesis testing problem

$$H_0 : \mu = \mu_0 \quad \text{versus} \quad H_A : \mu > \mu_0.$$

(a) Using the t-statistics $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$, we can reject H_0 with significant level α when the observed value t of T satisfies $t > t_{\alpha,n-1}$. (b) Alternatively we can construct the p-value $p^* = P(Y \ge t)$ with Y of t-distribution having $(n-1)$ degrees of freedom. Because of the relation "$p^* < \alpha \Leftrightarrow t > t_{\alpha,n-1}$," we can reject H_0 when $p^* < \alpha$.

Power of Test

Type II error and power. What is the probability that we incorrectly accept H_0 when it is actually false? Such probability β is called the probability of **type II error**. Then the probability $(1-\beta)$ of complement is known as the **power** of the test, indicating how *correctly* we can reject H_0 when it is actually false. Again, consider the problem of hospital workers subject to a radiation exposure. Given the current estimate $S = s$ of standard deviation and the current sample size $n = n_1$, the t-statistic $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$ can be approximated by $N(\delta, 1)$ with $\delta = \dfrac{\mu - \mu_0}{s/\sqrt{n_1}}$.

Example 4. Suppose that the true population mean is $\mu = 5.1$ (versus the value $\mu_0 = 5.4$ in our hypotheses). Then we can calculate the power of the test with $\delta \approx -2.11$ as follows.

Solution. (i) In the two-sided hypothesis testing, we reject H_0 when $|T| > t_{0.025,27} = 2.052$. Therefore, the power of the test is $P(|T| > 2.052) \approx 0.523$

(ii) In the one-sided hypothesis testing, we reject H_0 when $T < -t_{0.05,27} = -1.703$. Therefore, the power of the test is $P(T < -1.703) \approx 0.658$.

This explains why we could not reject H_0 in the two-sided hypothesis testing. Our chance to detect the falsehood of H_0 is only 52%, while we have 66% of the chance in the one-sided hypothesis testing.

Effect of sample size. For a fixed significance level α of your choice, the power of the test increases as the sample size n increases. In the two-sided hypothesis testing discussed above, we could recommend to collect additional data to increase the power of the test. But how many additional data do we need? Here is one possible way to calculate a desirable sample size n: In the two-sided hypothesis testing, the power $(1 - \beta)$ of the test is approximated by

$$P(|T| > t_{\alpha/2,n-1}) \geq P(Y > t_{\alpha/2,n-1} - |\delta|)$$

with a random variable Y having the t-distribution with $(n - 1)$ degrees of freedom. Given the current estimate $S = s$ of standard deviation and the current sample size n_1, we can achieve the power $(1 - \alpha/2)$ of the test by increasing a total sample size n and consequently satisfying $|\delta| \geq 2t_{\alpha/2,n_1-1}$.

Example 5. Suppose that we know the current estimate $\mu = 5.1$ of population mean and the current estimate $s = 0.7524$ of population standard deviation. In the above example of radiation exposure of hospital workers with $\mu_0 = 5.4$ in the hypothesis, calculate the recommended sample size for the study.

Solution. The sample size n can be calculated as

$$n \geq \left(\frac{2t_{\alpha/2,n_1-1}s}{|\mu - \mu_0|}\right)^2 = \left(\frac{2t_{0.025,27} \times 0.7524}{|5.1 - 5.4|}\right)^2 = 105.9.$$

What to do when rejected? When the null hypothesis H_0 is rejected, it is reasonable to find out the confidence interval of the population mean μ. The following table shows the confidence interval we can construct when your null hypothesis is rejected.

Alternative	When to reject H_0?	$(1-\alpha)$-level CI
$H_A : \mu \neq \mu_0$.	$\lvert T\rvert > t_{\alpha/2,n-1}$ †	$\left(\bar{X} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}},\ \bar{X} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}\right)$
$H_A : \mu > \mu_0$.	$T > t_{\alpha,n-1}$ †	$\left(\bar{X} - t_{\alpha,n-1}\frac{S}{\sqrt{n}},\ \infty\right)$
$H_A : \mu < \mu_0$.	$T < -t_{\alpha,n-1}$ †	$\left(-\infty,\ \bar{X} + t_{\alpha,n-1}\frac{S}{\sqrt{n}}\right)$

† $T = \frac{\bar{X}-\mu_0}{S/\sqrt{n}}$ is the test statistic, and α is the significant level.

Assignment No.7

Supplementary Readings.

SS: Chapter 4: Student t Distribution.

Chapter 5: Sampling Distribution of Variance, Case Where Population Variance is unknown.

Chapter 6: Confidence Interval for Means.

Chapter 7: Statistical Hypotheses, Tests of Hypotheses and Significance, Type I and Type II Errors, Level of Significance, One-Tailed and Two-Tailed Tests, P Value, Special Tests of Significance for Small Samples (Mean).

TH: Section 4.1–4.4.

Elliot A. Tanis and Robert V. Hogg, *A Brief Course in Mathematical Statistics.* Prentice Hall, NJ.

WM: Section 8.1–8.6, 9.1–9.4, and 10.1–10.4

Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, and Keying Ye, *Probability & Statistics for Engineers & Scientists*, 9th ed. Prentice Hall, NJ.

Problem 1. Construct a 95% two-sided confidence interval for the population mean in each of the following studies.

(a) A sample of 31 data observations has a sample mean $\bar{X} = 53.42$ and a sample standard deviation $S = 3.05$.

(b) A random sample of 41 glass sheets is obtained and their thicknesses are measured. The sample mean is $\bar{X} = 3.04$mm and the sample standard deviation is $S = 0.124$mm.

(c) A random sample of 16 one-kilogram sugar packets is obtained and the actual weights of the packets are measured. The sample mean is $\bar{X} = 1.053$kg and the sample standard deviation is $S = 0.058$kg.

Problem 2. An experimenter is interested in the hypothesis testing problem

$$H_0:\ \mu = 430.0 \quad \text{versus } H_A:\ \mu \neq 430.0,$$

where μ is the population mean of breaking strength of a bundle of wool fibers. Suppose that a sample of $n = 20$ wool fiber bundles is obtained and their breaking strengths are measured.

(a) For what values of the t-statistic does the experimenter *accept* the null hypothesis with significance level $\alpha = 0.10$?

(b) For what values of the t-statistic does the experimenter *reject* the null hypothesis with significance level $\alpha = 0.01$?

Suppose that the sample mean $\bar{X} = 436.5$ and the sample standard deviation is $S = 11.90$. Is the null hypothesis accepted or rejected with $\alpha = 0.10$? With $\alpha = 0.01$?

Problem 3. An experimenter is interested in the hypothesis testing problem

$$H_0:\ \mu = 0.065 \quad \text{versus } H_A:\ \mu > 0.065$$

where μ is the population mean of the density of a chemical solution. Suppose that a sample of $n = 61$ bottles of the chemical solution is obtained and their densities are measured.

(a) For what values of the t-statistics does the experimenter *accept* the null hypothesis with significance level $\alpha = 0.10$?

(b) For what values of the t-statistics does the experimenter *reject* the null hypothesis with significance level $\alpha = 0.01$?

Suppose that the sample mean $\bar{X} = 0.0768$ and the sample standard deviation is $S = 0.0231$. Is the null hypothesis accepted or rejected with $\alpha = 0.10$? With $\alpha = 0.01$?

Problem 4. A consumer agency suspects that a pet food company may be underfilling packages for one of its brands. The package label states "1200 grams net weight," and the president of the company claims the average weight is greater than the amount stated. For a random sample of 31 packages collected by the agency, the sample mean of the weights is $\bar{X} = 1182$ grams and the sample standard deviation is $S = 21$.

(a) Construct your hypothesis testing problem to find out the evidence that the pet food company is underfilling the packages.

(b) Complete your analysis, and present your statistical finding if any. And state your conclusion.

Problem 5. A machine is set to cut metal plates to a length of 44.350mm. The length of a random sample of 24 metal plates have a sample mean of $\bar{X} = 44.364$mm and a sample standard deviation of $S = 0.019$mm. Is there any evidence that the machine is miscalibrated?

Problem 6. A hair tonic for balding men and women is advertised as producing a reduction in hair loss of at least 25%. A consumer interest group ran an experiment with 23 volunteers and despite some unfortunate side effects, hair loss reductions with a sample mean $\bar{X} = 22.8\%$ and a sample standard deviation of $S = 8.72\%$ were observed. What evidence does the consumer interest group have that the advertised claim is false?

Problem 7. A regional office of the Internal Revenue Service randomly distributes returns to be audited to the pool of auditors. Over the thousands of returns audited last year, the average amount of extra taxes collected was \$356 per audited return. One of the auditors is suspected of being too lenient with persons whose returns are being audited. For a random sample of 30 of the returns audited by this person last year, an average of \$322 in extra taxes was collected, and the sample standard deviation is \$90. Briefly describe your hypothesis testing and your statistical finding, and give your conclusion.

Computer project. Continue from assignment No.6 for each of the following two case studies, and complete the hypothesis test regarding the study question associated with the experiment. Your report must include:

(a) Null and alternative hypothesis for the population mean μ;

(b) test statistic, critical value, and p-value;

(c) the result of hypothesis test with the choice of significance level 0.05;

(d) 95% and 99% confidence interval for μ;

The report should conclude with your answer to the study questions.

Case study 1, continued from No.6. In making aluminum castings, an average of 3.5 ounces per casting must be trimmed off and recycled as a raw material. A new manufacturing procedure has been proposed to reduce the amount of aluminum that must be recycled in this way. For a sample of 12 castings made with the new process, the following are the weights of aluminum trimmed and recycled.

4.2	2.0	3.6	3.8	2.6	3.5
3.2	2.6	2.5	1.7	3.3	4.8

Study question: What do you find regarding the evidence that the new process reduces the amount of trimmed aluminum?

Case study 2, continued from No.6. A chocolate manufacturer claims that at the time of purchase by a consumer the age of its product is less than 30 days. In an experiment to test this claim a random sample of 40 chocolate are found to have ages at the time of purchase. Download the data set "`chocolate.txt`" from the course website.

Study question: With this data how do you feel about the manufacturer's claim?

Answers

Problem 1.

(a) $53.42 \pm \frac{(2.042)(3.05)}{\sqrt{31}} = (52.30,\ 54.54)$

(b) $3.04 \pm \frac{(2.021)(0.124)}{\sqrt{41}} = (3.00,\ 3.08)$

(c) $1.053 \pm \frac{(2.131)(0.058)}{\sqrt{16}} = (1.022,\ 1.084)$

Problem 2. (a) The null hypothesis is accepted when $|T| \le 1.729$.

(b) The null hypothesis is rejected when $|T| > 2.861$.

Since we obtain $T = \frac{436.5-430.0}{11.90/\sqrt{20}} \approx 2.443$, the null hypothesis is rejected with significance level $\alpha = 0.10$, but it is not rejected with significance level $\alpha = 0.01$. In this case, you may conclude that the evidence against the null hypothesis is moderately significant.

Problem 3. (a) The null hypothesis is accepted when $T \le 1.296$.

(b) The null hypothesis is rejected when $T > 2.390$.

Since we obtain $T = \frac{0.0768-0.065}{0.0231/\sqrt{61}} \approx 3.99$, the null hypothesis is rejected with significance level $\alpha = 0.01$. Thus, the evidence against the null hypothesis is significant.

Problem 4. (a) To find out the evidence that the pet food company may be underfilling packages, the hypothesis testing problem becomes "$H_0:\ \mu = 1200$ versus $H_A:\ \mu < 1200$."

(b) The test statistic is $T = \dfrac{1182-1200}{21/\sqrt{31}} \approx -4.77$. If we choose the significance level $\alpha = 0.05$, then we have $-4.77 < -t_{0.05,30} = -1.697$; thus, we reject H_0. If we choose the significance level $\alpha = 0.01$, then we have $-4.77 < -t_{0.01,30} = -2.457$; thus, we reject H_0. In either case, the evidence is statistically significant.

Problem 5. The hypothesis test becomes "$H_0 : \mu = 44.350$ vs. $H_A : \mu \ne 44.350$." And we obtain the t-statistic $T = \frac{44.364-44.350}{0.019/\sqrt{24}} \approx 3.61$. With the choice of $\alpha = 0.01$, we have $|T| = 3.61 > t_{0.005,23} = 2.807$. Thus, we can conclude that the machine is miscalibrated.

Problem 6. The hypothesis test becomes "$H_0 : \mu = 25(\%)$ vs. $H_A : \mu < 25$." And we obtain the t-statistic $T = \frac{22.8-25}{8.72/\sqrt{23}} \approx -1.21$. With the choice of $\alpha = 0.05$, we have $T = -1.21 > -t_{0.05,22} = -1.717$. Thus, there is not a sufficient evidence that the advertised claim is false. (And we may need further research.)

Problem 7. To find out the evidence that this auditor is too lenient, we consider the hypothesis testing problem "$H_0:\ \mu = 356$ versus $H_A:\ \mu < 356$." The test statistic is $T = \dfrac{322-356}{90/\sqrt{30}} \approx -2.07$. Then $-2.07 < -t_{0.05,29} = -1.699$ (so you can reject H_0 if you choose $\alpha = 0.05$), but $-2.07 > -t_{0.01,29} = -2.462$ (that is, you cannot reject H_0 if you choose $\alpha = 0.01$). In this case you can say that the evidence is moderately significant.